University of Waterloo
Department of Electrical and Computer Engineering

E&CE 457 Applied Artificial Intelligence

RULE-BASED FUZZY EXPERT SYSTEMS

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1 Introduction

This project utilizes a fuzzy expert system in addressing the inverted pendulum problem since there is inherent uncertainty presented in the dynamics of the system. Design and implementation follows the provided handout closely. Pieces of the fuzzy system depicted below are implemented:

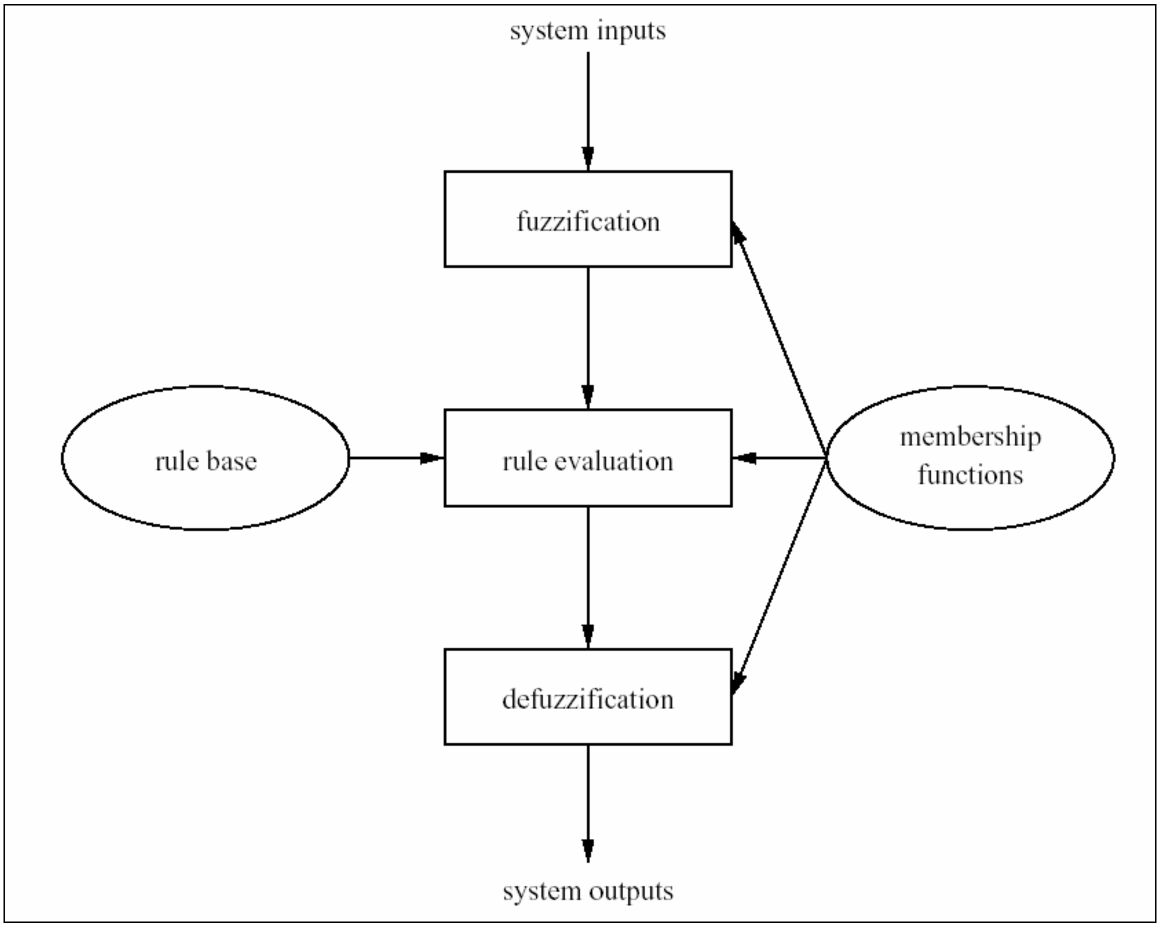


Figure 1: Diagram of Components Implemented

Crisp measurements are obtained from the inputs that are angular displacement and velocity of the inverted pendulum. These measurements are then "fuzzified" such that their degree of membership into their respective "fuzzy set" is determined. This is accomplished by considering the equations supplied in the handout that describe the behaviour of the system. After some careful manipulation, the optimal range for each fuzzy set is found. This shall be discussed in the next section.

Upon finding the degree of membership for angular displacement and velocity, a set of rules are then asserted to evaluate the resulting force that is applied on the block to counteract the falling pendulum. Several rule-based alternatives have been considered; however, we have chosen the 7x7 Rubric that will form the basis of our evaluation. This will also be explained in the next section.

Once the results are obtained, a process of defuzzification occurs at which the output, which is our applied force, is translated into a crisp value. The strength of the rule is dictated by the premises considered. Recall that the premises, or antecent, are degrees of membership of the inputs in the fuzzy set.

To simulate the inverted pendulum system, an initial value for angular displacement, velocity, and applied force is specified. Using this information, the resulting angular acceleration is calculated (with equations stated in the handout). An arbitrary length of time, or “time sample”, is specified to give the resulting angular velocity and displacement. With our fuzzy expert system, the force applied is determined. This process is cycled and its behaviour is observed. At this point, some tweaking of fuzzy sets was performed to improve the effectiveness of the system.

2 Discussion of Control Strategy

Control strategy is especially important for an effective fuzzy expert system that we strive to achieve. A mix of theory and intuition is combined in creating a control system that will provide the best possible performance.

2.1 Fuzzy Sets

Fuzzification is a process of computing the degree of membership of each current input value to the appropriate fuzzy set. This is required since inferencing (rule evaluation) requires some degree of membership in the premise of rules. Matlab was used to obtain a general "feel" of the numbers to expect.

2.1.1 Design of Fuzzy Sets

The behaviour of the system can be *generally* described by Fx in this next equation:

$$F_x \quad = \quad m\ddot{X} + \frac{mL}{2}\ddot{\theta}\cos\theta - \frac{mL}{2}\dot{\theta}^2\sin\theta \qquad (1)$$

Consequently, the acceleration of the block, X, is eliminated by rearranging and inserting this equation into (1):

$$M\ddot{X} = f_x - F_x \qquad (2)$$

Moreover, angular acceleration can be determined by the following equation and again, inserting into (1):

$$\dot{x}_2 \quad = \quad \frac{2g\sin x_1 - mL/[2(m+M)]x_2^2\sin(2x_1) - 2\cos x_1 u/(M+m)}{4L/3 - mL\cos^2\theta/(M+m)} \qquad (3)$$

Finally, the following Matlab m-file was created:

```
m = 2;
M = 8;
L = 1;
g = 9.8;
u = 0;

for i = 1 : 1 : 180,
    for j = 1 : 1 : 15,
        theta = (i-90)*pi/180;
        theta1 = j;
        theta2 = ( 2*g*sin(theta) - m*L/2/(m+M)*theta1^2*sin(2*theta) -
2*cos(theta)*u/(M+m)  ) / (4*L/3 - m*L*cos(theta)^2/(M+m));
        Fx(i,j) = (M/(m+M)) * (m/M*u + m*L/2*theta2*cos(theta) -
m*L/2*theta1^2*sin(theta));
    end
end
surf(Fx);

title('Simulation of Inverted Pendulum (u=0)');
xlabel('theta1 - angular velocity [rads/sec]');
ylabel('theta - angular displacement [rads]');
zlabel('Fx [N]');
```

Figure 2: Matlab m-file to Simulate Behaviour of Inverted Pendulum

This allows us to observe a 3-dimensional representation of the behaviour of the system at certain instances of angular displacement and velocity. Furthermore, the shape suggests possible maximum values that are essential in formulating the needed fuzzy sets. Several versions of the system at different forces applied, *u*, have been generated. These are shown below:

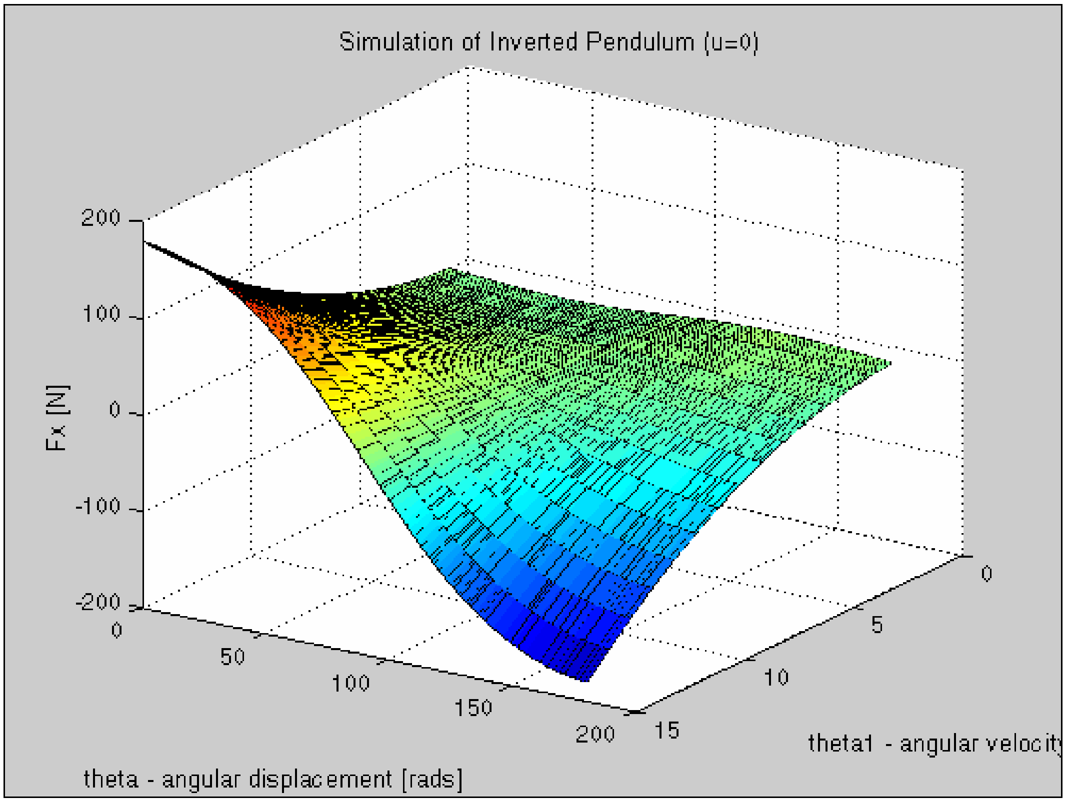


Figure 3.1 Behaviour of Inverted Pendulum at u = 0

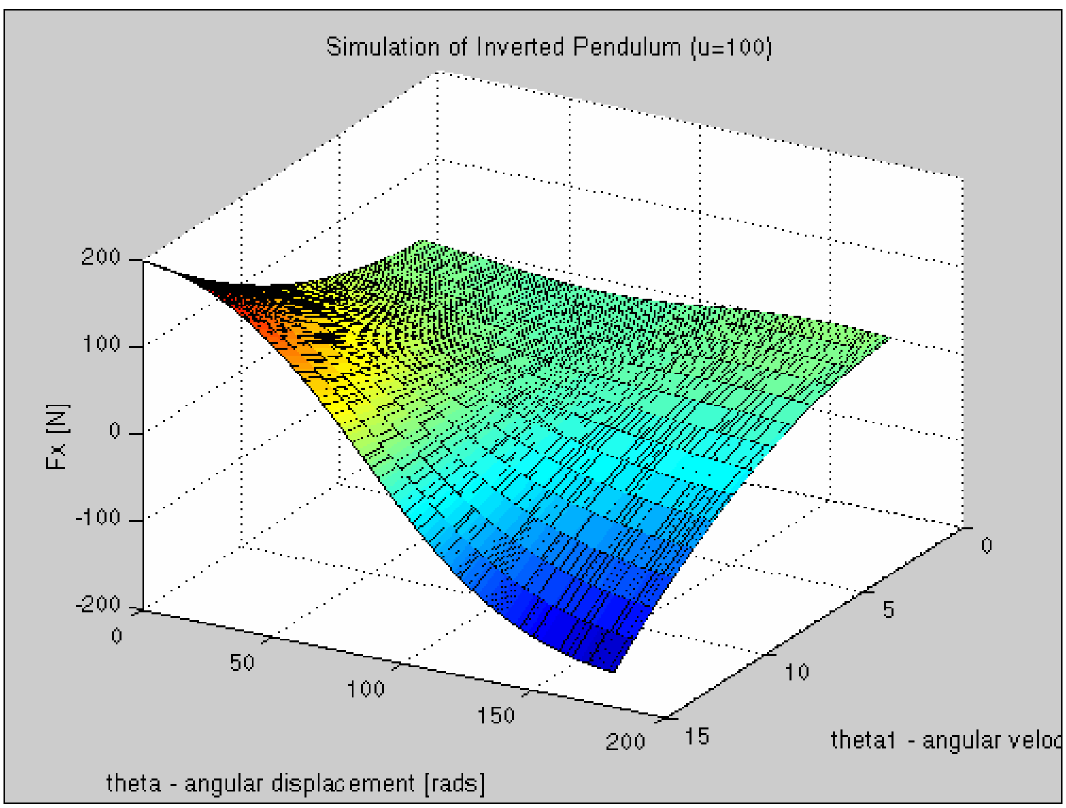


Figure 3.2 Behaviour of Inverted Pendulum at u = 100

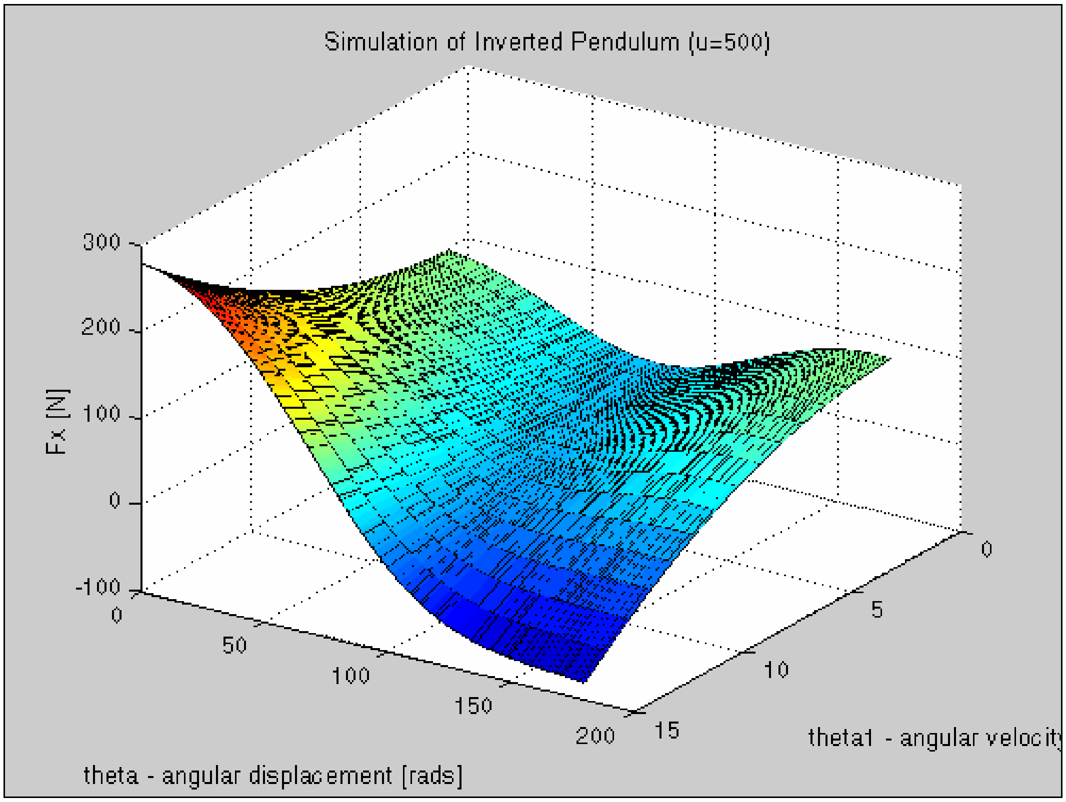


Figure 3.3 Behaviour of Inverted Pendulum at u = 500

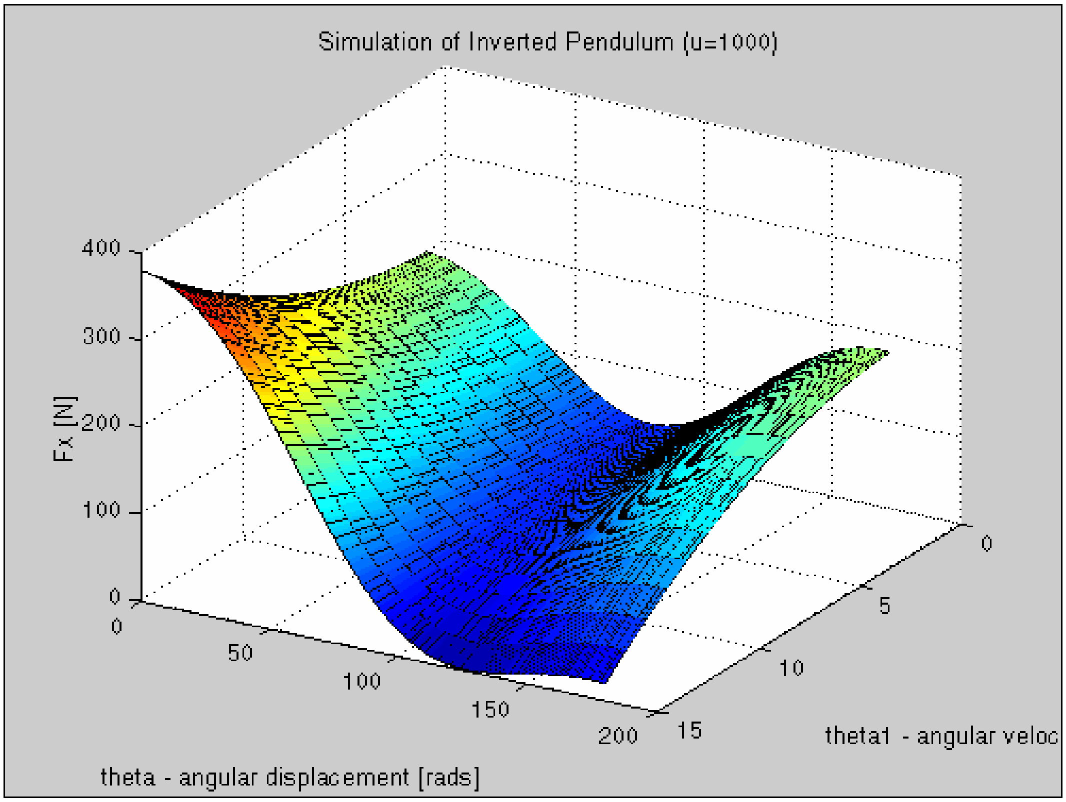


Figure 3.4 Behaviour of Inverted Pendulum at u = 1000

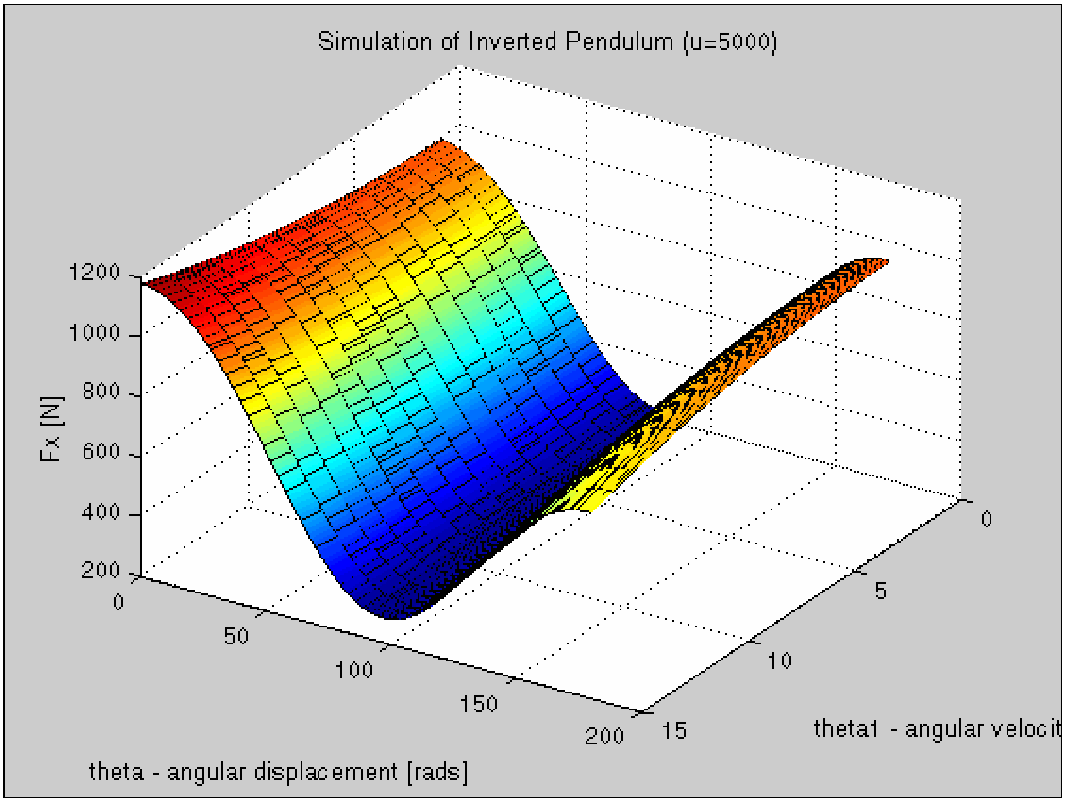


Figure 3.5 Behaviour of Inverted Pendulum at u = 5000

Initially, when there is no applied force (u = 0), Fx exerted on the bottom of the pendulum exhibits a bell shaped well that behaves as expected. For example, at zero velocity and at an angle of 90 degrees, Fx is evaluated to be zero. Oppositely, as angular displacement and velocity increases, the *magnitude* of Fx increases which intuitively agrees with the expected behaviour. In addition, as the applied force rises, the rise and drop of the contour becomes more dramatic and indicates the proportional nature of Fx.

Judging from the progressive dip of the mesh as the force applied rises, it may be *beneficial to increase the sensitivity of those regions that are near the upright position of the pendulum.* This implies that the triangular fuzzy sets should be situated closer together for smaller angular displacement.

2.1.2 Implementation of Fuzzy Set

The proposed fuzzy set comprises of overlapping triangles of seven degrees of intensity that are NL, NM, NS, ZE, PL, PM, and PS. In the event that the sensor detects an angle, or velocity, within the region of overlap, several approaches have been considered in resolving the ambiguity from being part of two sets – not quite one or the other. For instance, a certain angle may fall between a portion of NM (0.8) and NS (0.3). Thus, this can be handled in three ways.

First, we may simply consider only the set with the greatest degree of membership; in which case, we shall select NM(0.8) as a premise for our rule base. (Although we are neglecting NS, which essentially "linearizes" our system, our initial simulations show that this approach is still possible since the system is *quite* sure that it is NM and response is gauged according to this assumption.)

Second, we may evaluate the "Center of Gravities" of the two overlapping fuzzy sets and select the larger degree of membership of the resulting fuzzy set, or have them evaluated independently by the next method.

Third, we may keep either fuzzy set independent and assert the rules as usual, but for conflicting conclusions we may choose to defuzzify according to the aforementioned "Center of Gravity" or simply, choose the maximum of the rule strengths.

In any case, so long as the format of the premise is satisfied, that is, a single fuzzy set is represented for angular displacement and velocity, our proposed rule base will function accordingly.

2.1.3 Remarks

From our Matlab analysis, we have established a general range of permissible values for our initial fuzzy sets that are 0 - 10 rad/s and 0 - 7000 N for force applied. However, after numerous trials, the fuzzy sets have been tweaked. We found that the system would perform at its best from 0 – 2pi rad/s and 0 - 5000 N for force applied. As well, the peaks at the end have been slightly skewed to "strengthen" the degree of membership once the range has been determined. For example, for an *incredibly fast velocity* (NL), one would not expect the membership to diminish beyond the peak of its corresponding fuzzy set.

As well, we have tuned our system for subtle changes in angle since any amount of force applied for a displacement of 90 degrees would not render the pendulum back into the upright position. Consequently, applying a force that is too great would lead to overshoot. Further mathematical analysis may be formulated in the future to optimize this system. Actual definition of fuzzy sets for simulation purposes does not overlap.

2.2 Rule Base

The rule base for the fuzzy expert system follows the general form IF (PREMISE) THEN CONCLUSION as described in the lectures. The dissection of the rule tells us that each premise and conclusion has an associated degree of membership.

The strength of each rule is computed, which is the minimum of the antecents. However, there may be conflicting rules that must be resolved. Thus, defuzification by "Center of Gravity" may be utilized to resolve membership of output in multiple sets to a crisp value.

2.2.1 Design of Rule Base

A 7x7 Rule Evaluation Rubric is chosen as the rule base. Evaluation is straightforward. In coming up with the rubric, seven degrees of angular displacement and velocity have been compared and a resulting applied force was assigned (also varying by 7 degrees). Naturally, we began with the most obvious comparisons along the diagonals of the grid -- a ZE angular displacement and ZE angular velocity would evaluate to ZE force applied;

conversely, NL and NL respectively would equate to NL force applied. Next, comparisons with obvious "middle of the line" results were established -- NS and NL would give NM. For situations where there were no definite solutions, the value between must be interpolated. This is accomplished by simply considering the average between two adjacent centroids. The output is the resulting degree of membership from this process.

Other ideas that we have mustered up (but have not muddled with) were to have separate rules asserted for each angular displacement and each angular velocity. For instance, PL angular displacement would be evaluated to PL force applied while PS angular velocity would result in PS force applied. The average of the two conclusions PL and PS would be used to gauge the intensity of the force applied; in which case, would be PM. In many ways, this is similar to the reasoning behind the method stated above except the "rubric" would be dynamically conceived by the application rather than having it explicitly stated.

2.2.2 Implementation of Rule Base

Please refer to the diagram below (*Figure 5*). Empty spaces are interpolated from adjacent results.

2.2.3 Remarks

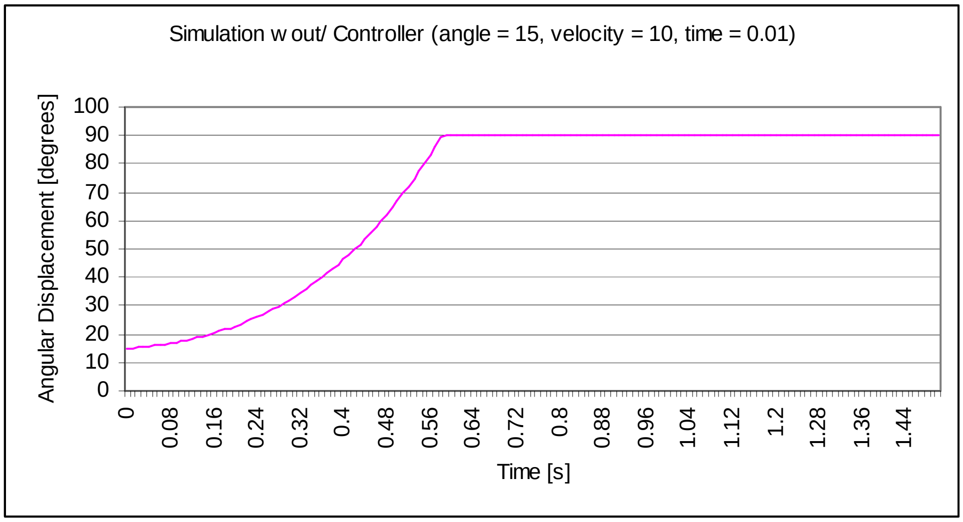
Given additional time, perhaps other rules may be explored. So long as the rules cover the possible cases, and evaluates to a seemingly appropriate output, then the rule base should be applicable. Rather, the problem that is presented is a matter of optimality and may involve sophisticated mathematical analysis to determine the best possible number of fuzzy states, coverage, and rule-base. Otherwise, system performance will depend on our individual dexterity for trial and error.

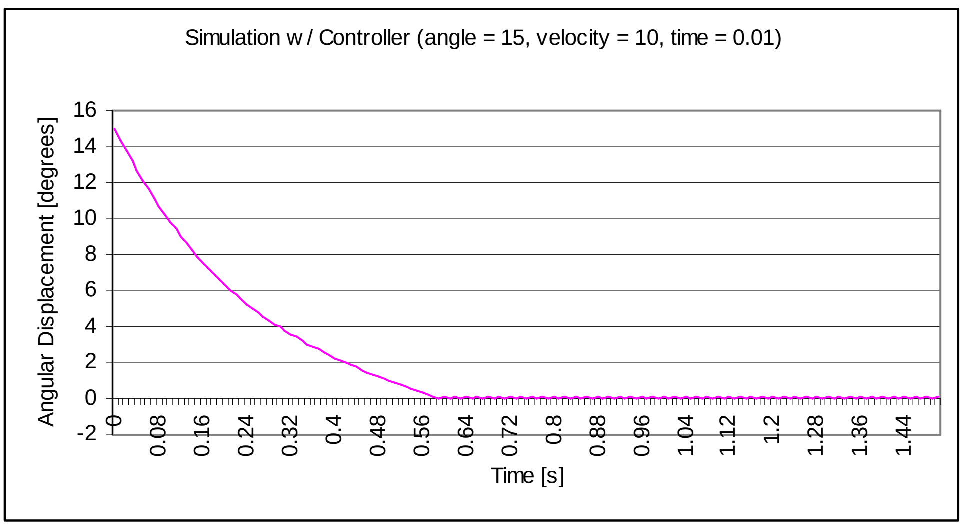
In reality, our implementation is optimized for minor changes in angle rather than broad sweeping motions. Hence, we can generalize large angular velocity as just exerting a sufficiently large force in the same direction to counteract the motion.

3 Simulation of Controller

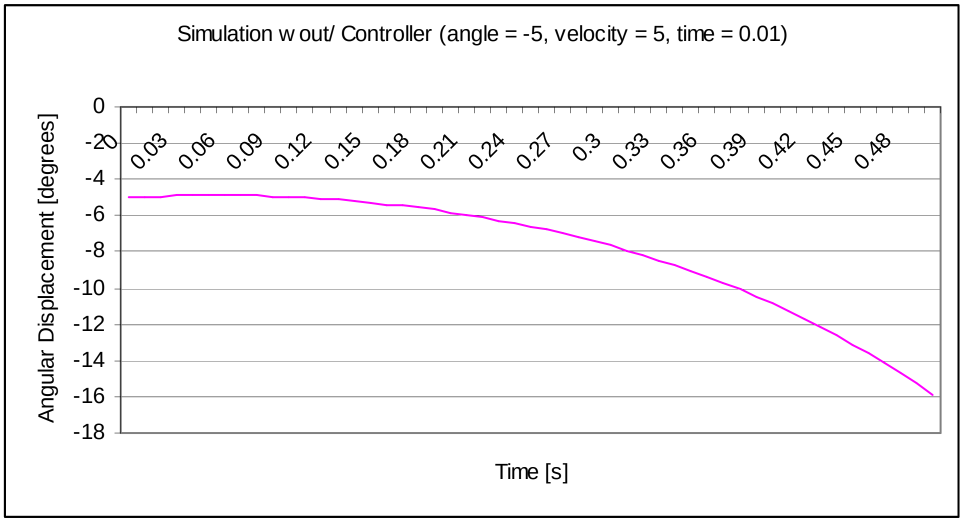
A simulation of the fuzzy controller in action is displayed below.

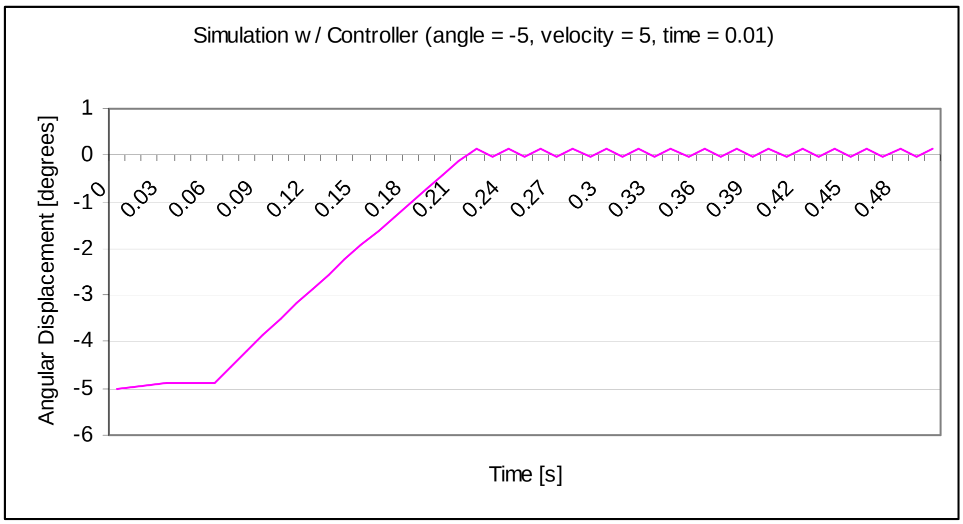
Case 1: Plot of System Dynamics for Initial Angle = 15 and Velocity = 10





Case 2: Plot of System Dynamics for Initial Angle = -5 and Velocity = 5





Case 3: Plot of System Dynamics for Initial Angle = -45 and Velocity = -20